

Student Workbook

Finding Limits: Numerical and Graphical Approaches

Name: _____

Date: _____

Period: _____

Learning Target: I can estimate limits using tables of values and graphs, including one-sided limits, and determine when a limit does not exist.

Vocabulary

Term	Meaning
Limit	The value $f(x)$ approaches as x gets arbitrarily close to a (from both sides).
One-Sided Limit	The value $f(x)$ approaches from only the left ($x \rightarrow a^-$) or only the right ($x \rightarrow a^+$).
DNE	"Does Not Exist" — used when a limit has no single value.

Key Concept: Estimating a Limit Numerically

METHOD Evaluate $f(x)$ at values very close to a from BOTH sides. If both sides approach the same number, that's the limit.

EXAMPLE 1
Estimate $\lim_{x \rightarrow 2} f(x)$ of $f(x) = x^2 + 1$.

x	1.9	1.99	2.01	2.1
$f(x)$	4.61	4.9601	5.0401	5.41

The limit appears to approach _____

Key Concept: When a Limit Does NOT Exist

A TWO-SIDED LIMIT EXISTS ONLY IF BOTH SIDES AGREE If $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$, then $\lim_{x \rightarrow a} f(x)$ DOES NOT EXIST (DNE).

EXAMPLE 2

A graph has a jump discontinuity at $x=1$: the left branch approaches $y=3$, the right branch approaches $y=5$.

Since $3 \neq 5$, $\lim_{x \rightarrow 1} f(x) = \underline{\hspace{2cm}}$

COMMON MISTAKE

A limit can exist even if $f(a)$ is UNDEFINED (like at a hole)! The limit only cares about the values NEAR a , not the value exactly AT a .

Key Concept: Holes vs. the Actual Function Value

EXAMPLE 3

A graph shows $f(x)$ approaching $y=6$ from both sides as $x \rightarrow 3$ (a hole), even though $f(3)$ is undefined.

$\lim_{x \rightarrow 3} f(x) = \underline{\hspace{2cm}}$ (the hole does NOT prevent the limit from existing)

Guided Practice — Part A: Estimate Using a Table (Polynomial)

- A1** Estimate $\lim_{x \rightarrow 3} f(x)$ of $f(x)=x^2-2$ using $x=2.9, 2.99, 3.01, 3.1$.

- A2** Estimate $\lim_{x \rightarrow 1} f(x)$ of $f(x)=2x+3$ using $x=0.9, 0.99, 1.01, 1.1$.

Guided Practice — Part B: Estimate Using a Table (Rational, with a Hole)

- B1** Estimate $\lim_{x \rightarrow 4} f(x)$ of $f(x)=(x^2-16)/(x-4)$ using a table.

- B2** Estimate $\lim_{x \rightarrow 5} f(x)$ of $f(x)=(x^2-25)/(x-5)$ using a table.

Guided Practice — Part C: Does the Limit Exist?

C1 Given $\lim_{x \rightarrow 2^-} f(x) = 4$ and $\lim_{x \rightarrow 2^+} f(x) = 4$, does $\lim_{x \rightarrow 2} f(x)$ exist? If so, what is it?

C2 Given $\lim_{x \rightarrow 3^-} f(x) = 6$ and $\lim_{x \rightarrow 3^+} f(x) = 9$, does $\lim_{x \rightarrow 3} f(x)$ exist?

Guided Practice — Part D: Reading Graphs

D1 A graph shows $f(x)$ approaching $y=2$ from both sides as $x \rightarrow 5$, but $f(5)=7$ (a hole with a separate point). What is $\lim_{x \rightarrow 5} f(x)$?

D2 A graph shows a vertical asymptote at $x=-2$: $f(x) \rightarrow +\infty$ from the right, $f(x) \rightarrow -\infty$ from the left. Does $\lim_{x \rightarrow -2} f(x)$ exist?

BEFORE YOU MOVE ON

Check with a partner: did you check BOTH sides (left and right) before concluding whether the limit exists?